CLAIMS

What is claimed is:

1. A method to determine a numerical solution of a linear system of equations representing a physical entity, comprising:

generating a mesh representation of the physical entity, the mesh representation comprising mesh elements;

computing a linear system matrix A of coefficients by computing interactions between simple functions defined over sets of mesh elements;

partitioning the mesh representation into a plurality of partitions separated by partition boundaries; and

computing a preconditioner for the coefficient matrix A that is compatible with the linear system of equations and that provides at least basis function support over at least two mesh elements, where coupling of the preconditioner between partitions is only through basis functions at the partition boundaries.

2. A method as in claim 1, where the preconditioner is itself a valid solution to the same set of physical equations that govern the full linear system.

3. A method as in claim 1, where computing a preconditioner operates to compute a preconditioning matrix K where partition boundaries are constrained to coincide with the edges of mesh elements, and to compute mesh element interactions using reduced coupling.

4. A method as in claim 3, where mesh element interactions between basis functions are computed only for half functions within the same partition, where a half function denotes the function over any one of multiple mesh elements for which it is defined, and where the interactions of basis functions crossing a partition boundary are computed separately for each of the half functions such that no interactions exist between basis function halves that are defined in separate ones of the partitions, and those basis functions completely within a partition, referred to as interior elements, interact only with other interior elements and with boundary element halves within the same partition.

5. A method as in claim 4, further comprising sorting indices of basis functions in the matrices A and K so that all internal elements appear first, grouped according to their respective partitions, followed by all boundary elements, and where a resulting preconditioning matrix K for n partitions has the form:

$$K = \begin{bmatrix} [Ka_1] & & & & \\ & [Ka_2] & & & [Kb] \\ & & \ddots & & \\ & & & [Ka_n] & \\ & [Kc] & & & [Kd] \end{bmatrix},$$

where the sub matrix Ka is the block diagonal matrix created by the union of the matrices of internal element interactions Ka_1 through Ka_n, Kd represents the interactions between the boundary elements, and Kb and Kc are the interactions between the internal and boundary elements.

6. A method as in claim 5, further comprising iteratively solving a system of equations $Ax=f$ using the linear system matrix A, a vector f of boundary conditions on each mesh element and the preconditioner matrix K to provide an approximate solution x.

7. A method as in claim 6, where the linear system matrix A is partitioned in the same manner as the preconditioner using the same partitions, separate partitions, or a combination of the same and separate partitions.

8. A method as in claim 6, where iteratively solving comprises operating a conjugate gradient iterative solver.

9. A signal bearing medium tangibly embodying a program of machine-readable instructions executable by a digital processing device to perform operations to compute a numerical solution of a linear system of equations representing a physical entity, the operations comprising:

generating a mesh representation of the physical entity, the mesh representation comprising mesh elements;

computing a linear system matrix A of coefficients by computing interactions between simple functions defined over sets of mesh elements;

partitioning the mesh representation into a plurality of partitions separated by partition boundaries; and

computing a preconditioner for the coefficient matrix A that is compatible with the linear system of equations and that provides at least basis function support over at least two mesh elements, where coupling of the preconditioner between partitions is only through basis functions at the partition boundaries.

10. A signal bearing medium as in claim 9, where the preconditioner is itself a valid solution to the same set of physical equations that govern the full linear system.

11. A signal bearing medium as in claim 9, where computing a preconditioner operates to compute a preconditioning matrix K where partition boundaries are constrained to coincide with the edges of mesh elements, and to compute mesh element interactions using reduced coupling.

12. A signal bearing medium as in claim 11, where mesh element interactions between basis functions are computed only for half functions within the same partition, where a half function denotes the function over any one of multiple mesh elements for which it is defined, and where the interactions of basis functions crossing a partition boundary are computed separately for each of the half functions such that no interactions exist between basis function halves that are defined in separate ones of the partitions, and those basis functions completely within a partition, referred to as interior elements, interact only with other interior elements and with boundary element halves within the same partition.

13. A signal bearing medium as in claim 12, further comprising sorting indices of basis functions in the matrices A and K so that all internal elements appear first, grouped according to their respective partitions, followed by all boundary elements, and where a resulting preconditioning matrix K for n partitions has the form:

$$K = \begin{bmatrix} [Ka_1] & & & & \\ & [Ka_2] & & & [Kb] \\ & & \ddots & & \\ & & & [Ka_n] & \\ & [Kc] & & & [Kd] \end{bmatrix},$$

where the sub matrix Ka is the block diagonal matrix created by the union of the matrices of internal element interactions Ka_1 through Ka_n, Kd represents the interactions between the boundary elements, and Kb and Kc are the interactions between the internal and boundary elements.

14. A signal bearing medium as in claim 13, further comprising iteratively solving a system of equations $Ax=f$ using the linear system matrix A, a vector f of boundary conditions on each mesh element and the preconditioner matrix K to provide an approximate solution x.

15. A method as in claim 14, where the linear system matrix A is partitioned in the same manner as the preconditioner using the same partitions, separate partitions, or a combination of the same and separate partitions.

16. A signal bearing medium as in claim 14, where iteratively solving comprises operating a conjugate gradient iterative solver.

17. A digital processing system operable to compute a numerical solution of a linear system of equations representing a physical entity, comprising:

a generator to output a mesh representation of the physical entity, the mesh representation comprising mesh elements;

a first computation function to compute a linear system matrix A of coefficients by computing interactions between simple functions defined over sets of mesh elements;

a partitioner to partition the mesh representation into a plurality of partitions separated by partition boundaries; and

a second computation function to compute a preconditioner for the coefficient matrix A that is compatible with the linear system of equations and that provides at least basis function support over at least two mesh elements, where coupling of the preconditioner between partitions is only through basis functions at the partition boundaries.

18. A digital processing system as in claim 17, where the computed preconditioner is itself a valid solution to the same set of physical equations that govern the full linear system.

19. A digital processing system as in claim 17, where said second computation function operates to compute a preconditioning matrix K where partition boundaries are constrained to coincide with the edges of mesh elements, and to determine mesh element interactions using reduced coupling.

20. A digital processing system as in claim 19, where said second computation function computes mesh element interactions between basis functions only for half functions within the same partition, where a half function denotes the function over any one of multiple mesh elements for which it is defined, and where the interactions of basis functions crossing a partition boundary are computed separately for each of the half functions such that no interactions exist between basis function halves that are defined in separate ones of the partitions, and those basis

functions completely within a partition, referred to as interior elements, interact only with other interior elements and with boundary element halves within the same partition.

21. A digital processing system as in claim 20, where said second computation function operates to sort indices of basis functions in the matrices A and K so that all internal elements appear first, grouped according to their respective partitions, followed by all boundary elements, and where a resulting preconditioning matrix K for n partitions has the form:

$$K = \begin{bmatrix} [Ka_1] & & & & \\ & [Ka_2] & & & [Kb] \\ & & \ddots & & \\ & & & [Ka_n] & \\ & [Kc] & & & [Kd] \end{bmatrix},$$

where the sub matrix Ka is the block diagonal matrix created by the union of the matrices of internal element interactions Ka_1 through Ka_n, Kd represents the interactions between the boundary elements, and Kb and Kc are the interactions between the internal and boundary elements.

22. A digital processing system as in claim 21, further comprising an iterative solver operable to solve a system of equations $Ax=f$ using the linear system matrix A, a vector f of boundary conditions on each mesh element and the preconditioner matrix K to provide an approximate solution x.

23. A method as in claim 22, where the linear system matrix A is partitioned in the same manner as the preconditioner using the same partitions, separate partitions, or a combination of the same and separate partitions.

24. A digital processing system as in claim 22, where said iterative solver comprises a conjugate gradient iterative solver.